The Effects on S, T, and U from Higher-Dimensional Fermion Representations

Hong-Hao Zhang, Yue Cao, and Qing Wang
Department of Physics, Tsinghua University, Beijing 100084, China

Inspired by a new class of walking technicolor models recently proposed using higher-dimensional technifermions, we consider the oblique corrections from heavy non-degenerate fermions with two classes of higher-dimensional representations of the electroweak gauge group itself. One is chiral SM-like, and the other is vector-like. In both cases, we obtain explicit expressions for S, T, U in terms of the fermion masses. We find that to keep the T parameter ultraviolet-finite there must be a stringent constraint on the mass non-degeneracy of a heavy fermion multiplet.

PACS numbers: 12.15.Lk, 12.38.Bx, 12.90.+b

I. INTRODUCTION

Despite its tremendous success, the standard model (SM) has several drawbacks. On the one hand, the Higgs particle has not yet been found in experiments; on the other hand, the SM suffers the hierarchy problem and triviality from a theoretical point of view. Thus, the SM may not be correct, or at least it is just an effective theory at the electroweak scale. There are many new physics possibilities beyond the SM. Although we do not know whether nature really behaves like one of them or not, we can estimate their effects on the current electroweak precision measurements. Peskin and Takeuchi's S, T, U-formalism is a practical way to do this job [1]. Since the current SM parameter fits indicate that S and T are small negative numbers, and U is also close to zero [2], those new physics models which give large positive contributions to S and T are presumably excluded. Thus, the oblique correction parameters S, T, U are often used to judge whether a new model is compatible with experiments or not. If the SM is not a full theory, there will be new heavy particles above the electroweak scale. Provided the new particles feel the electroweak interactions, they should give corrections to S, T, U whether they are fermions, scalars, or gauge bosons.

Recently, there has been increasing interest in a new class of walking technicolor models, using technifermions with higher-dimensional, rather than fundamental, representations of the technicolor gauge group [3]. Their walking dynamics feature can avoid unacceptably large flavor changing neutral currents. If these models were true, in general it will be also possible for the presence of heavy fermions with higher-dimensional representations of the electroweak gauge group $SU(2)_L \times U(1)_Y$ itself. Of course, these particles could give corrections to S, T, and U. In an earlier paper by Dugan and Randall [4], the effects to the S parameter from general fermion represention of $SU(2)_L \times SU(2)_R \times U(1)_Y$ has been considered assuming a strict custodial $SU(2)_C$ symmetry. Later, the corrections to S, T, U and also to triple-gauge-vertices from a heavy non-degenerate fermion doublet has been estimated respectively [1, 5].

In this paper, we will calculate the corrections to S, T, and U from two classes of higher-dimensional fermion representations of $SU(2)_L \times U(1)_Y$ itself. One is the SM-like chiral type, in which right-handed fermions are singlets, while left-handed fermions form a multiplet of the $SU(2)_L \times U(1)_Y$ group. The other is the vector-like case, in which the left and right-handed fermion multiplets transform the same way under the electroweak group. In the following, the strict custodial symmetry will be relaxed to an approximate symmetry so as just to keep the T parameter ultraviolet-finite. In each case, we obtain a mass constraint on a fermion multiplet to satisfy this demand. At the end of the paper a brief concluding remark is given.

II. THE SM-LIKE CHIRAL REPRESENTATIONS

Consider a SM-like heavy fermion multiplet with $N = 2j+1$ dimensions and with quantum numbers of $SU(2)_L \times U(1)_Y$ as

$$\psi_L = \begin{pmatrix} \psi_j \\ \psi_{j-1} \\ \vdots \\ \psi_{-j} \end{pmatrix}_L \sim (2j+1, Y)\,,$$
$$\psi_{l,R} \sim (1, Y+l)\,, \qquad (l = j,\, \dots,\, -j)\,. \tag{1}$$

For simplicity, we restrict this electroweak multiplet to be a color and technicolor singlet. If it is also a color multiplet, the effect on the result is just a multiplying factor of the number of colors. In general, this N-plet is non-degenerate, and we denote their masses m_l where the subscript l runs from $-j$ to j. They couple to the electroweak gauge bosons via

$$\frac{e}{\sqrt{2}s}(W^+_\mu J^\mu_+ + W^-_\mu J^\mu_-) + \frac{e}{cs}Z_\mu(J^\mu_3 - s^2 J^\mu_Q) + eA_\mu J^\mu_Q\,, \tag{2}$$

where $c \equiv \cos\theta_W$, $s \equiv \sin\theta_W$, and

$$\begin{aligned}
J^\mu_+ &= \sum_{l=-j}^{j-1} \sqrt{(j-l)(j+l+1)}\bar{\psi}_{l+1,L}\gamma^\mu\psi_{l,L}\,,\\
J^\mu_- &= \sum_{l=-j+1}^{j} \sqrt{(j+l)(j-l+1)}\bar{\psi}_{l-1,L}\gamma^\mu\psi_{l,L}\,,\\
J^\mu_3 &= \sum_{l=-j}^{j} l\,\bar{\psi}_{l,L}\gamma^\mu\psi_{l,L}\,, \qquad J^\mu_Q = \sum_{l=-j}^{j}(l+Y)\bar{\psi}_l\gamma^\mu\psi_l\,.
\end{aligned} \tag{3}$$

By computing the vacuum polarization amplitudes for the N-plet fermion, we obtain their contributions to the oblique correction parameters S, T, and U. As expected, we find that S and U are always ultraviolet-finite, and they are

$$S \equiv 16\pi[\Pi'_{33}(0) - \Pi'_{3Q}(0)] = \frac{1}{3\pi}\sum_{l=-j}^{j}\left[l^2 - 2\,l\,Y\,\log\left(\frac{m_l^2}{\mu^2}\right)\right]\,, \tag{4}$$

$$U \equiv 16\pi[\Pi'_{11}(0) - \Pi'_{33}(0)] = \frac{4}{\pi}\left[\sum_{l=-j}^{j-1}\frac{(j-l)(j+l+1)}{2}f_1(m^2_{l+1}, m^2_l) - \sum_{l=-j}^{j}\frac{l^2}{6}\log\left(\frac{m_l^2}{\mu^2}\right)\right]\,, \tag{5}$$

where μ is a mass scale parameter and the function f_1 is defined as

$$f_1(m^2_{l+1}, m^2_l) \equiv \int_0^1 dx\, x(1-x)\log\left[\frac{xm^2_{l+1} + (1-x)m^2_l}{\mu^2}\right]\,. \tag{6}$$

But the T parameter can be generally ultraviolet-divergent, and the result is

$$\begin{aligned}
T &\equiv \frac{4\pi}{s^2c^2m_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)]\\
&= \frac{1}{4\pi s^2c^2m_Z^2}\left[A\Big[\frac{1}{\epsilon} - \gamma - \frac{1}{2} + \log(4\pi)\Big] + \sum_{l=-j}^{j}2\,l^2m_l^2\log\left(\frac{m_l^2}{\mu^2}\right) - \sum_{l=-j}^{j-1}(j-l)(j+l+1)f_2(m^2_{l+1}, m^2_l)\right]\,,
\end{aligned} \tag{7}$$

where the divergent term has been dimensionally regularized by setting $\epsilon = (4-d)/2$, and the coefficient A and the function f_2 are respectively

$$A \equiv \sum_{l=-j}^{j}(j^2 + j - 3\,l^2)m_l^2\,, \tag{8}$$

$$f_2(m^2_{l+1}, m^2_l) \equiv \int_0^1 dx[xm^2_{l+1} + (1-x)m^2_l]\log\left[\frac{xm^2_{l+1} + (1-x)m^2_l}{\mu^2}\right]\,. \tag{9}$$

In order to avoid the unacceptable disastrous divergence, the coefficient A must be zero. This relation gives a strong constraint on the mass non-degeneracy of fermion multiplets. Since a small value of the T parameter is related to the approximate custodial $SU(2)_C$ symmetry; if $A \neq 0$, the custodial symmetry will be disastrously violated. Thus we call $A = 0$ the custodial symmetry soft-breaking condition. For $j = 1/2$, (*i.e.* for a fermion doublet,) this condition is satisfied automatically. For $j = 1$, the constraint is $m_1^2 + m_{-1}^2 = 2m_0^2$. For a general j, an interesting particular example respecting this constraint is $m_l^2 = m^2 + l\Delta m^2$, for l running from $-j$ to j.

III. THE VECTOR-LIKE REPRESENTATIONS

Next, we consider a $(2j+1)$-dimensional vector-like fermion multiplet as

$$\psi_L = \begin{pmatrix} \psi_j \\ \psi_{j-1} \\ \vdots \\ \psi_{-j} \end{pmatrix}_L \sim (2j+1, Y) \; , \qquad \psi_R = \begin{pmatrix} \psi_j \\ \psi_{j-1} \\ \vdots \\ \psi_{-j} \end{pmatrix}_R \sim (2j+1, Y) \; . \tag{10}$$

The interaction between these fermions and the electroweak gauge bosons is of the same form as Eq.(2), but now

$$\begin{aligned} J_+^\mu &= \sum_{l=-j}^{j-1} \sqrt{(j-l)(j+l+1)}\bar{\psi}_{l+1}\gamma^\mu\psi_l \; , \\ J_-^\mu &= \sum_{l=-j+1}^{j} \sqrt{(j+l)(j-l+1)}\bar{\psi}_{l-1}\gamma^\mu\psi_l \; , \\ J_3^\mu &= \sum_{l=-j}^{j} l \; \bar{\psi}_l\gamma^\mu\psi_l \; , \qquad J_Q^\mu = \sum_{l=-j}^{j} (l+Y)\bar{\psi}_l\gamma^\mu\psi_l \; . \end{aligned} \tag{11}$$

Likewise, we compute their contributions to S, T, and U resulting in

$$S \;=\; -\frac{2\,Y}{3\pi}\sum_{l=-j}^{j} l \; \log(\frac{m_l^2}{\mu^2}) \; , \tag{12}$$

$$U \;=\; \frac{4}{\pi}\Bigg[\sum_{l=-j}^{j-1}(j-l)(j+l+1)\bigg(f_1(m_{l+1}^2, m_l^2) - \frac{1}{2}f_3(m_{l+1}^2, m_l^2)\bigg) + \sum_{l=-j}^{j}\frac{l^2}{6}[1 - 2\log(\frac{m_l^2}{\mu^2})]\Bigg] \; , \tag{13}$$

$$T \;=\; \frac{1}{4\pi s^2 c^2 m_Z^2}\Bigg[B\big[\frac{1}{\epsilon} - \gamma - \frac{1}{2} + \log(4\pi)\big] - 2\sum_{l=-j}^{j-1}(j-l)(j+l+1)f_2(m_{l+1}^2, m_l^2)\Bigg] \; , \tag{14}$$

where the functions f_1 and f_2 have been defined in Eqs.(6) and (9), and the function f_3 and the coefficient B are respectively

$$f_3(m_{l+1}^2, m_l^2) \equiv \int_0^1 dx \frac{x(1-x)m_{l+1}m_l}{x m_{l+1}^2 + (1-x)m_l^2} \; , \tag{15}$$

$$B \equiv \sum_{l=-j}^{j-1}(j-l)(j+l+1)(m_{l+1} - m_l)^2 \; . \tag{16}$$

In this case, the custodial symmetry soft-breaking condition $B = 0$ implies all the m_l's must be equal, *i.e.*, this vector-like multiplet must be degenerate, otherwise the custodial symmetry will be unacceptably broken. But a mass-degenerate vector-like fermion multiplet gives zero contribution to S, T, and U. Thus, any custodial-symmetry preserved vector-like fermion representations have no effect on the oblique correction parameters.

IV. A CONCLUDING REMARK

In this paper, we have obtained the one-loop corrections to S, T, and U from two classes of higher-dimensional fermion representations. When taking the fermion masses to be equal, our expression for the S parameter coincide with the PDG's result of S for degenerate fermions [2]. When taking $j = 1/2$ in the SM-like case, our expressions for S, T, and U are exactly those given in Ref. [1].

We have shown that for the chiral case, in order to keep T ultraviolet-finite, there must be a constraint on the mass non-degeneracy of the chiral multiplet. While for the vector-like case, this constraint becomes even more stringent, and it demands that vector-like multiplets must be degenerate, which further implies that vector-like fermions cannot give any contributions to S, T, U as long as an approximate custodial symmetry is imposed. These mass constraints may be potentially useful for some model-building considerations.

Although the case of SM-like chiral representations we considered above is just a special case where right-handed fermions are all weak-singlets, it is sufficient to illustrate the point. A generalization of this work to more general chiral representations might be straightforward.

Acknowledgments

We would like to thank Prof. H. J. He and Dr. J. K. Parry for helpful discussions. This work is supported by the National Natural Science Foundation of China, and the Fundamental Research Foundation of Tsinghua University.

---

[1] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990);
M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992).
[2] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).
[3] F. Sannino and K. Tuominen, Phys. Rev. D **71**, 051901 (2005);
D. K. Hong, S. D. H. Hsu and F. Sannino, Phys. Lett. B **597**, 89 (2004);
D. D. Dietrich, F. Sannino and K. Tuominen, Phys. Rev. D **72**, 055001 (2005);
D. D. Dietrich, F. Sannino and K. Tuominen, Phys. Rev. D **73**, 037701 (2006);
N. D. Christensen and R. Shrock, Phys. Lett. B **632**, 92 (2006).
[4] M. J. Dugan and L. Randall, Phys. Lett. B **264**, 154 (1991).
[5] T. Inami, C. S. Lim, B. Takeuchi and M. Tanabashi, Phys. Lett. B **381**, 458 (1996);
M. Tanabashi, Lecture given in *Frontiers of Particle Physics 2006: Beyond the Standard Model*, Beijing (2006).